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Modeling and Control of Uncertain Systems with Applications to Air Force Problems

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Abstract

This final report summarizes the achievements of our research program, and describes in details the research results obtained during the past three years. The primary goal of this research program is investigation of novel approaches to robust-control-oriented system identification techniques and robust feedback control system design methods that solve typical Air Force problems. This goal has been accomplished successfully focusing on gap-metric, and ν-metric uncertain systems described by normalized coprime factors, and on rotating stall and surge control for axial flow compressors as application platform, that was demonstrated in the 9 journal publications, two book chapters, and 14 conference papers, including those accepted for publication during the past three years. Several algorithms were developed to identify the nominal model using frequency domain data with quantification of the unmodeled dynamics in $\mathcal{H}_\infty/\mathcal{L}_\infty$ norm, that are shown to converge in presence of the worst-case noise contaminated in experimental data. Control of gap-metric, and ν-metric uncertain systems was studied in connection with multiplicative/relative perturbations in general coprime factors, and $\mathcal{H}_\infty$ loopshaping design method was generalized to $\mathcal{L}_\infty$ loopshaping that apply to those systems involving uncertainties in normalized coprime factors with bounded $\mathcal{L}_\infty$-norm. Moreover an effective algorithm was developed for model reduction of observer-based controllers. The PI has also strived to enhance the application part of this research program, and has spent two summers in Wright Laboratory at Wright-Patterson Air Force base, and at Eglin Air Force Base. In close collaborations with the control group in Wright Laboratory of the Wright-Patterson Air Force base (led by Dr. Siva Banda), application research in our program has focused on compressor control that are essential to the improvement of aeroengines and performance of jet airplanes. Our contributions in this research area included a bifurcation approach to stabilization of flow instabilities such as rotating stall and surge that are shown to be effective for the low order Moore-Greitzer model. With the success of this research program, we are confident that our control group is well positioned, and ready to make further contributions to the Air Force mission.

Objectives

There was a significant change for the objectives of the research effort in the last two years of our research program. With the emphasis on nonlinear robust control, and applications to solve Air Force problems, our program made a transition from modeling and identification of linear uncertain systems to those of nonlinear systems. In particular, active control for rotating stall and surge in axial flow compressors has been the focus in our research effort for the past two years, and will be the sole objective as well for our research group in the next a few years. The PI discussed this change with Dr. Jacobs in the 1995 AFOSR Contractor Meeting held in Minneapolis, Minnesota, and has been collaborating with the research group, led by Dr. Siva Banda, in WPAFB since then. Although our program has focused on rotating stall and surge control for the past two years, modeling and control of uncertain systems was the main objective for the first year of our research program with emphasis on novel identification and control algorithms for flexible structures. The work on identification and control of linear uncertain systems was also carried over to the subsequent two years.

1 Introduction

Modeling and control of uncertain systems have received great attention in the past decade. This research area is spurred by high performance and reliability requirements for flight control systems where large uncertainties are involved due to different flight conditions. The development of $\mathcal{H}_\infty$, and ℓ^1 optimal control in the eighties, and emergence of identification in $\mathcal{H}_\infty$, and ℓ^1 in the nineties have advanced feedback control system design techniques greatly. Various Air Force problems have provided application platforms for modeling and control of uncertain systems.

Our research program began in August of 1994 focusing on novel approaches to robust-control-oriented system identification techniques and robust feedback control system design methods that solve typical Air Force problems. Through hard work, and close collaborations with Air Force Research Laboratories, we have successfully completed most of the proposed research plan under a much smaller budget[1]. Our major contributions are integration of identification and control, and bifurcation stabilization with applications to compressor control. To be more specific, Several algorithms were developed to identify the nominal model using frequency domain data with quantification of the unmodeled dynamics in $\mathcal{H}_\infty/\mathcal{L}_\infty$ norm. The identified uncertain systems are described by normalized coprime factors related to gap-metric, and ν-metric uncertain systems. Both iterative and non-iterative algorithms were proposed that are shown to be convergent in presence of the worst-case

[1] Our budget was cut by 53.7%.

noise contaminated in experimental data. Control of gap-metric, and ν-metric uncertain systems was studied in connection with multiplicative/relative perturbations in general coprime factors. Moreover $\mathcal{H}_\infty$ loopshaping design method was generalized to $\mathcal{L}_\infty$ loopshaping that apply to those systems involving uncertainties in normalized coprime factors that are unbounded in gap-metric, and ν-metric, but small in $\mathcal{L}_\infty$-norm. Because $\mathcal{H}_\infty/\mathcal{L}_\infty$ loopshaping design methods often result in observer-based feedback compensators, controller reduction was investigated that yields an effective model reduction algorithm for observer-based controllers. The same algorithm applies to coprime factors reduction of the plant model that has *a priori* multiplicative/relative error bound in $\mathcal{H}_\infty$-norm.

The PI has strived to enhance the application part of our research program, and has spent two summers in Wright Laboratory at Wright-Patterson Air Force base, and at Eglin Air Force Base. In close collaborations with the control group led by Dr. Siva Banda in Wright Laboratory of the Wright-Patterson Air Force base, application research in our program has focused on compressor control that is essential to the improvement of aeroengines and performance of jet airplanes. Our initial contributions in this regard included a bifurcation approach to stabilization of flow instabilities such as rotating stall and surge that was shown to be effective for the low order Moore-Greitzer model. Specifically, local output feedback stabilization with smooth nonlinear controllers was studied for parameterized nonlinear systems for which the linearized system possesses either a simple zero eigenvalue, or a pair of imaginary eigenvalues, and the bifurcated solution is unstable at the critical value of the parameter. Stabilizability conditions were obtained for both stationary and Hopf bifurcations where the critical mode (or modes) of the linearized system is observable or unobservable. It was shown that either linear controllers or quadratic controllers are adequate for bifurcation stabilization. Moreover stabilizing controllers were characterized in explicit form that can be used in synthesis. Applications to rotating stall and surge control were investigated with throttle as actuator that are effective for the Moore-Greitzer model and extend the stable operating range of the compression system. Several feedback control laws were proposed to stabilize the operating point at the peak pressure rise of the axial flow compressors. The first one employed pressure rise as the output measurement. Both linear and nonlinear feedback controllers were obtained for rotating stall control. The second one employed mean flow rate as the output measurement that again is effective for suppression of rotating stall. The success of rotating stall control for the third order Moore-Greitzer model led naturally to the problem of surge control. A linear state feedback control was employed for surge control. It was shown that for any compressor parameters set, there exist state feedback control laws that stabilize the critical rotating stall mode, and avoids surge entirely. While this is a good news, the required feedback gains can be excessively large. Thus several test functions were derived for a given state feedback gain to determine whether or not the surge can be

avoided for a given range of the parameter set. If the surge is not avoidable, the procedure to design stabilizing feedback gains was also obtained. Hence for the third order Moore-Greitzer model, our results are quite satisfactory.

Regarding education, this research program has made a great effort to train graduate students. In the past three years, the PI offered a special topics course on bifurcation analysis and compressor control. He taught one special topics class on robust identification in $\mathcal{H}_\infty$, and is currently teaching the same class again in Fall of 1997. More than 30 students are benefited through the special topics class on robust identification in $\mathcal{H}_\infty$. Based on the research results obtained in the past several years, and lecture notes prepared for the special topics class, the PI wrote the first draft of a research book on system identification in $\mathcal{H}_\infty$ that will be submitted for publication in the near future. This book will be beneficial to a more broad audience in the control community. During the past three years, the PI also offered a special topics course on bifurcation analysis and compressor control that helped train graduate students work in the area of rotating stall and surge control. Moreover with the help of the ECE Department at LSU, this program was able to support one graduate student to complete his Ph.D Dissertation and another to complete his M.S. Thesis, both on modeling and control of uncertain systems. Presently there is another Ph.D student working on his Ph.D Dissertation specialized on compressor control. He is expected to graduate May 1998. Considering that the scale of our group is rather small, it is a quite accomplishment in education for our research program.

This program would like to thank Dr. Marc Jacobs for giving us the opportunity to work under the Program of Dynamics and Control, AFOSR, and to contribute to Air Force missions. This final report summarizes our achievements in both research and education during the past three years, and describes in details the results obtained by our research program. Our research findings will be reported in the next section.

2 Accomplishments/New Findings

Since the inception of our program in Sept. 1, 1994, the research effort of our program has focused initially on modeling and control of uncertain systems that is theory oriented, and transited later to rotating stall and surge control for axial flow compresssors that is application oriented. Because axial flow compressors are the vital part of the aeroengine, and rotating stall and surge limits effectively the aeroengine performance, our transition to application oriented research on compressor control is in the interest of the Air Force, which was approved by Program Manager Dr. Marc Jacobs. It was also realized later that the our work on modeling and control of uncertain systems has applications

to compressor control. In the next four subsections our accomplishments and new findings will be described in more details.

2.1 Modeling of Uncertain Systems

Modeling of uncertain systems is a continuation of system modeling and identification. The inadequacy of the conventional identification algorithms for high performance control system design has been recognized and has recently received great attention [30]. Two basic methodologies approach the modeling of uncertain systems. One is a stochastic approach based on conventional identification technique to quantify the model uncertainty in a probabilistic framework. The other is a deterministic approach which aims to quantify the model uncertainty in $\mathcal{H}_\infty$ norm or ℓ^1 norm compatible to robust control. Control oriented identification is deterministic in nature. A worst case formulation was proposed first by Helmicki, Jacobson and Nett [14] which has led to rapid development in the past a few years. Although several effective algorithms were developed for identification in $\mathcal{H}_\infty$ which include linear algorithms [15], two-stage nonlinear algorithms [14, 12], and interpolatory algorithms [7, 13], the form of uncertainties is restricted to be additive. It is clear that such simple structure of uncertain systems limits greatly the applicability to robust control system design. The reason lies in the fact that the robust stability condition for additive uncertain systems may conflict with performance requirement of the feedback control system design. Thus even though the uncertain systems are described in frequency domain, it could be difficult to integrate identification and control under the same framework of $\mathcal{H}_\infty$. Another consideration is that the additive uncertainty may not be suitable for modeling of lightly damped systems such as flexible structures due to large abrupt variations in its frequency response that are difficult to approximate with small additive error bound. In order to circumvent the above mentioned problems, our program has focused on identification of coprime factor uncertainties. The results included two effective algorithms for identification of the nominal model, and quantification of the unmodeled dynamics described by its normalized coprime factors. mOne of the proposed algorithms was shown to be convergent in the presence of the worst-case noise that will be discussed in more detail next.

The system in consideration is linear, shift-invariant, and possibly infinite dimensional. Its transfer function matrix exists and is denoted by $P(z)$. It is assumed that $P(z)$ has size $p \times m$ with $p \geq m$, and admits a right normalized coprime factorization

$$P(z) = N(z)[D(z)]^{-1}, \quad N^{\sim}N + D^{\sim}D \equiv I_m, \tag{1}$$

with $N(z)$ and $D(z)$ both analytic in the region $|z| \geq 1$. Let the generalized plant of $P(z)$ be defined

by

$$G(z) := \begin{bmatrix} D(z) \\ N(z) \end{bmatrix} = \sum_{k=0}^{\infty} g_k z^{-k}, \;\; g_k \in \mathbf{R}^{q\times m}, \;\; q = p + m. \tag{2}$$

It is assumed that $G \in \mathcal{S}$ where $\mathcal{S}$ is a strict subset of $\mathcal{H}_\infty$ that characterizes the *a priori* information of the true unknown plant. The system identification problem can be described as follows.

Assume: that the generalized plant $G(z) \in \mathcal{S}$ that is both inner and outer;

Given: a finite set of frequency response measurement samples of the plant

$$E_k = (I_p + \eta_k)\, P_k, \;\; P_k = P(e^{j2k\pi/N}), \;\; \eta \in \mathcal{N}_\epsilon, \tag{3}$$

where $0 \le k \le N-1$ and $\mathcal{N}_\epsilon := \left\{\eta_k : \; \|\eta_k P_k (I_p + E_k^H E_k)^{-1/2}\|_{\ell^\infty} \le \epsilon\right\}$;

Find: $P_{id}(z)$, the identified model, together with an upper bound on the uncertainty

$$e_\nu(\mathcal{S}) \;=\; \sup_{\eta\in\mathcal{N}_\epsilon,\, G\in\mathcal{S}} \|(I_p + PP^\sim)^{-1/2}(P - P_{id})(I_m + P_{id}^\sim P_{id})^{-1/2}\|_\infty \tag{4}$$

that converges to zero as $N \to \infty$ and $\epsilon \to 0$ where $\|\cdot\|_\infty$ denotes $\mathcal{L}_\infty$-norm.

It is noted that in the above identification problem, the error quantification $e_\nu(\mathcal{S})$ is the same as ν-metric if in addition the condition

$$\kappa = \mathrm{wno}\det(I_m + P^\sim P_{id}) + \mu(P_{id}) - \mu(P) = 0 \tag{5}$$

is true where $\mu(\cdot)$ denotes the number of strictly unstable poles, and $\mathrm{wno}(\cdot)$ the winding number of standard Nyquist contour. A recent result of [33] indicates that even if $\kappa > 0$, controllers having the same stabilizing property as that for gap-metric uncertainties exist. Hence the above identification problem fits robust control well. Moreover frequency response of the normalized coprime factors is much more smooth than that of the plant model.

Several difficulties are also involved with this particular identification problem. The first is the lack of the experimental data on normalized coprime factors of the unknown plant, even though the objective is to minimize the uncertainty described by normalized coprime factors. The second is the quantification error where the true plant and its normalized coprime factors are unknown. The final is the winding number condition that is hard to take into consideration in the development of the corresponding identification algorithm.

Our research program proposed two different algorithms that successfully resolve the above mentioned identification problem. The first algorithm was based on discrete Fourier analysis and balanced stochastic truncation. Using the property of normalized coprime factors, we were able to convert frequency response measurement samples of the plant into experimental data of the normalized coprime

factors. The discrete Fourier analysis were then applied to obtain an approximate spectral function generated by the normalized coprime factors. The low order normalized coprime factors were finally obtained through the use of generalized balanced stochastic truncation. The combination of DFA and BST yields an effective algorithm for modeling of lightly damped systems in form of normalized coprime factor uncertainties that were demonstrated through numerical examples. Moreover worst-case identification errors were quantified and shown to converge to zero as the number of frequency response samples increases to infinity and the noise level decreases to zero.

A second algorithm is iterative in nature. Although the first algorithm is effective, it has some problem in handling large quantity of the experimental data. Thus a simpler algorithm was proposed. It was shown in our work that with fixed order nominal model, the identification problem can be converted into an equivalent nonconvex optimization. In order to make it feasible numerically, iterative scheme was employed in searching the local optimum by fixing part of the unknown parameters that makes the optimization convex. Thus LMI toolbox in MATLAB can be used to compute the identified model that converges to local optimum. Quadratic norm was also employed as the error measurement that yields fast convergence, and efficient computation. The large quantity of the experimental data in the process of identification can be treated without problem. However it was unsuccessful to obtain the *a priori* error bound, as well as to prove its robust convergence.

During the three year of our research program, our program also completed the work on interpolation algorithm for continuous-time systems in collaboration with researchers elsewhere. Although this is a generalization of our earlier work on interpolation of discrete-time systems, the generalization to continuous-time systems is not trivial. Contribution was also made to linear algorithms that were proposed by other researchers in the same field. This included improvement of the performance for linear algorithms with more tight error bound. The combination of the linear algorithm with balanced truncation yields an effective approach to obtain low order state-space models. Regarding education, one Ph.D student, and one M.S. student were trained. The PI is also in the process of preparing a research book for the research field of robust identification in $\mathcal{H}_\infty$ that was taught in a special topics class for graduate students at LSU, and is currently used again for the same course.

In summary, this program has achieved most of the objectives as proposed in our research plan, although there was not enough time for us to investigate identification problem using the time-domain data due to the transition of our research program to application research on compressor control.

2.2 Control of Uncertain Systems

Robust control of uncertain systems has been extensively studied in the past decade. Due to the lack of effective synthesis algorithms for parametric uncertain systems, $\mathcal{H}_\infty$ control methodology has

gained more popularity. Recent work on loopshaping [8, 25] has made $\mathcal{H}_\infty$ control more appealing to practicing engineers. Loopshaping has its root in the classical lead-lag compensator design [35], and thus it is a natural extension of classical control design to the multivariable control systems. It assumes that the uncertainties are described by normalized coprime factors of the nominal plant, and they are equivalent to gap metric uncertainty [10]. Robust control of such uncertain systems has been studied in great extent that leads to the development of $\mathcal{H}_\infty$ loopshaping design methodology. Our proposed research along this direction was related to the integration with identification of the uncertain system. As discussed in the previous section, our identified uncertainty may not be gap-metric bounded, or even ν-metric bounded that is the case if $\kappa > 0$. See (5). Therefore, uncertain systems described by normalized coprime factors with stable perturbations are not suitable for our purposes. Moreover the computation of gap metric uncertainty involves two-block $\mathcal{H}_\infty$ optimization that is difficult to obtain. Hence more general type of uncertain systems is needed for better integration of identification and control. This motivated generalization of $\mathcal{H}_\infty$ loopshaping, and many related problems such as controller reduction as well. In the rest of the section we will present several results obtained by our research program.

Our first result was robust control of uncertain systems described by general coprime factor uncertain systems that are not necessarily normalized. Suppose that the true plant $P_t(s) = \tilde{M}_t^{-1}\tilde{N}_t$ with $\tilde{M}$ and $\tilde{N}$ stable that are not normalized. The uncertain system in consideration is represented by multiplicative/relative perturbations described by coprime factors $\tilde{M}$ and $\tilde{N}$, and has the form:

$$\tilde{G}_t = \begin{bmatrix} \tilde{M}_t & \tilde{N}_t \end{bmatrix} = \begin{bmatrix} \tilde{M} & \tilde{N} \end{bmatrix} \left(I_q + \Delta_{mul}\right) = \tilde{G}\left(I_q + \Delta_{mul}\right), \tag{6}$$

$$\tilde{G} = \begin{bmatrix} \tilde{M} & \tilde{N} \end{bmatrix} = \begin{bmatrix} \tilde{M}_t & \tilde{N}_t \end{bmatrix} \left(I_q + \Delta_{rel}\right) = \tilde{G}_t\left(I_q + \Delta_{rel}\right), \tag{7}$$

where $P = \tilde{M}^{-1}\tilde{N}$ is the nominal model, and $\Delta_{mul} \in \mathcal{H}_\infty$ and $\Delta_{rel} \in \mathcal{H}_\infty$ are bounded in $\mathcal{H}_\infty$-norm. The necessary and sufficient condition for stabilizability of such uncertain systems was derived that is identical to the stabilizability condition for the gap-metric uncertainty. This is surprising because the coprime factors considered in our uncertain systems are not normalized.

The above reported result has an important implication. A similar result holds for uncertainty described by coprime factors of the feedback controller with multiplicative/relative perturbations. If the feedback controller has an observer form, then coprime factors of the controller can be obtained through a particular general inverse of $\tilde{G}$ in $\mathcal{H}_\infty$. Because the uncertainty of the controller often results from controller reduction, it motivated us to investigate model reduction of observer-based controllers which were studied by the research group of Anderson [4, 22, 23] before us. But the problem was only partially solved. An inverse weighted balanced truncation method was proposed by the PI that results in *apriori* multiplicative/relative error bound in coprime factor of the plant

or observer-based controller. The model reduction algorithm has the same feature as that proposed in [25] in that the plant and controller are simultaneously reduced that is very different from that of [4, 22, 23]. If the observer-based controller is a normalized LQG controller, then it reduces to the result of controller reduction reported in [25]. Stability conditions of the closed-loop system consisting of full order plant and reduced order observer-based controller were obtained and are parallel to those in [25]. Since these conditions hold for coprime factors other than normalized, our results compliment the existing results in [25] on controller reduction. For normalized LQG control, our proposed inverse weighted balanced truncation yields a truncated controller that is again a normalized LQG controller. Model reduction for LQG controller was studied first by Jonckhere and Silverman [17]. Although it is tackled in [25], the resulting reduced order controller is not a normalized LQG controller. Our proposed algorithm for controller reduction gave a satisfactory solution to this problem.

Our next result is on the connection of coprime factor uncertain systems involving multiplicative/relative perturbations and that of gap-metric and ν-metric uncertainty. Because the robust stability condition for the family of uncertain systems in (6) and (7), as well as for feedback control systems consisting of full order plant and reduced order observer-based controller is identical to that for gap-metric uncertain systems, it raised question of the possible connection between these two different types of uncertainties. In collaboration with Dr. Li Qiu, the PI has proved several results that help understand better the uncertain systems described by coprime factors of the plant. It was discovered in the research course that the set of uncertain plants needs be enlarged to include $\mathcal{L}_\infty$ perturbations under some condition in order to be equivalent to gap-metric uncertainty. If instead, some winding number condition is satisfied, then the enlarged set of uncertain systems is equivalent to ν-metric uncertainty. By connecting the gap or the ν-gap with multiplicative/relative perturbations on coprime factors that are not necessarily normalized, more insight into the robust control theory was provided that made the theory more convenient and versatile, and paved the way for the extension of the theory to the cases when normalized coprime factorizations are not desirable, such as infinite dimensional systems [31, 11], or to cases when normalized coprime factorizations are not possible, such as systems with Banach input output spaces [28].

As indicated in the previous section, the uncertain systems identified from the proposed identification algorithms may not be in the form of gap metric, or even ν-metric. The use of $\mathcal{H}_\infty$ loopshaping method for robust control design becomes questionable. Hence our program also made an effort to generalize $\mathcal{H}_\infty$ loopshaping to those systems involving possible unstable perturbations described by normalized coprime factors. It was shown that the design methodology of $\mathcal{H}_\infty$ loopshaping can be generalized to $\mathcal{L}_\infty$ loopshaping for which the feedback controller may not stabilize the nominal model, but stabilize the true unknown plant. In particular, the feedback system for true unknown

system is stable, if the number of unstable poles of the feedback system for the nominal plant is precisely κ as defined in (5), and certain $\mathcal{L}_\infty$-norm is suitably small. Furthermore, all properties for $\mathcal{H}_\infty$ loopshaping are preserved with minor modifications. The work on $\mathcal{L}_\infty$ loopshaping facilitates greatly the integration of identification and control for robust control system design.

Our research program also investigated $\mathcal{H}_\infty$ loopshaping in the form of weighted mixed sensitivity minimization. It was motivated by parametrically mixed sensitivity studied in [19] where closed formulae were derived for $\mathcal{H}_\infty$ optimization. A more general problem of weighted mixed sensitivity minimization was considered in [20] where the choice of weighting functions and synthesis of feedback controllers were examined. A particular form was considered for weighted mixed sensitivity minimization and its equivalence to the $\mathcal{H}_\infty$ loopshaping proposed in [25] was shown. A free parameter $\lambda > 0$ was introduced to trade-off minimizations of sensitivity and complementary sensitivity in the $\mathcal{H}_\infty$ performance index that was referred to parametric $\mathcal{H}_\infty$ loopshaping. It was shown that the problem of parametric $\mathcal{H}_\infty$ loopshaping involves solving an $\mathcal{H}_\infty$ type ARE (algebraic Riccati equation). A formula was obtained for the optimal performance index that reduces to the explicit formula of [25] at $\lambda = 1$. However for $\lambda \neq 1$, the optimal performance index is given implicitly, and its computation is rather complicated. A modified Newton-Raphson method was proposed to compute the optimal performance index iteratively that is quadratically convergent. Similar to the $\mathcal{H}_\infty$ loopshaping, the result in this paper can be used for stabilization of uncertain systems where normalized coprime factors involve parametrically weighted $\mathcal{H}_\infty$ norm bounded perturbations. Controller reduction was investigated for suboptimal controllers without model reduction of the plant. Robust stability condition was established for the feedback system consisting of full order plant and reduced order controller, and an estimate was obtained for the associated $\mathcal{H}_\infty$ performance index. Our program also carried out for controller reduction of normalized LQG control initiated in [17]. Similar results were derived for its robust stability and $\mathcal{H}_\infty$ performance index. Our results apply to normalized LQG control that is a special case of the parametric $\mathcal{H}_\infty$ loopshaping.

In retrospect, this research program has obtained quite many research results during the past three years regarding robust control, although these results are more diversified and less focused than our work on identification of uncertain systems as reported in the previous section. This feature is also illustrated by our work on constrained optimal control and stability of time delay systems.

2.3 Integration of Identification and Control in Frequency Domain

A distinguished feature of our research program is the integration of identification and control for uncertain systems. In our work of modeling for uncertain systems, attention was always paid to the form and structure of uncertainties that are suitable for the employment of robust control methodol-

ogy. On the other hand in our work of control part, the uncertain models obtained from identification was also kept in mind. This strategy has worked well for our research on modeling and control of uncertain systems that results in better integration of identification and control for uncertain systems. For instance, for identification of coprime factor uncertainty which will be used for $\mathcal{H}_\infty$ or $\mathcal{L}_\infty$ loopshaping design at a later stage, weighting functions were introduced that yield desirable loopshape for the weighted plant before the application of any identification algorithm. Moreover the selection of weighting function also took the filtering of the experimental data into consideration so that less contaminated data can be used in the process of identification. Because the objective for filtering of the experimental data may differ from that of loopshaping, extra trade-off was necessary in determination of weighting functions. Such an issue was ignored in the past, but does have its role in integration of identification and control for uncertain systems. Another instance is in the control part where loopshaping design method was used for synthesis of feedback control system. Because the identified may not have the same number of unstable poles as that of the true plant, the winding number condition for the ν-metric is often violated. This motivated us to derive $\mathcal{L}_\infty$ loopshaping design method that treats the case $\kappa > 0$ as defined in (5).

The integration of identification and control for uncertain systems is especially important for light damped systems such as flexible structures where large number of modes are close to instability, and thus frequency response shows abrupt variations in short frequency intervals. Hence the identified model may have different number of poles on or close to the imaginary axis. It should be pointed out that in the work of [32, 33], the modes on the imaginary axis was not treated correctly. This problem was resolved in our work on identification and control for uncertain systems. Because our research papers regarding integration of identification and control were listed at the end of the previous two sections, those interested are referred to the previous two section for more details.

2.4 Bifurcation Stabilization and Compressor Control

For application research, the PI has strived to contribute to the DoD mission. In particular, the PI has spent two summers in the Air Force Laboratory at Wright-Patterson Air Force Base, and at Eglin Air Force Base. The PI tried to pursue the direction of robust adaptive control for missile autopilot control problem. But later, he was more convinced that compressor control had a more urgent need for Air Force due to his contact with Dr. Siva Banda who is the leader of the control group at Wright-Patterson Air Force Base. His vision and his excellent control group have given the PI great benefits when he spent the summer of 1995 there. Aften then the PI made an important transition from theory oriented research on modeling and control of uncertain systems to application oriented research on rotating stall and surge control for axial flow compressors.

Axial flow compressors are the vital part of turbine-based aeroengines. However, the engine performance is effectively reduced by rotating stall and surge in axial flow compressors, which are instabilities that arise in the unsteady fluid dynamics. Both these instabilities are disruption of the normal operating condition designed for steady and axisymmetric flow. Rotating stall is a severely non-axisymmetric distribution of axial flow velocity which manifests itself as a region of severely reduced flow that rotates at a fraction of the rotor speed. Prolonged operation under this condition may break rotor blades, and burn the turbine [25]. Surge, on the other hand, is an axisymmetric pumping oscillation which can cause flameout and thus engine damage as well. Both lead to large penalties in performance of aeroengines.

Our Methodology to Approach Compressor Control Problems

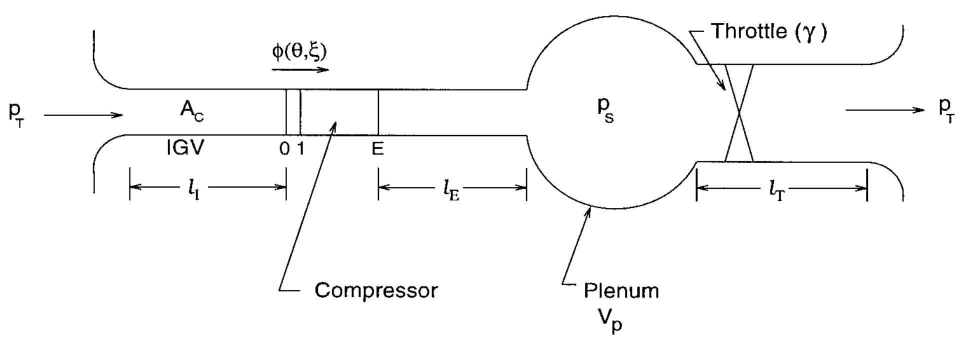
Since rotating stall and surge significantly limit the performance of turbine-based aeroengines, and have catastrophic consequences if occur in jet airplanes, compressor control becomes the priority in the list of research problems for AFOSR. Our research program, though small, shows the effort of AFOSR to resolve this important research problem in the near future. It is recognized in both the research community and AFOSR that employing feedback control to suppress rotating stall and surge is essential for extending compressor operating range and to improving performance of the future aeroengines. Hence the research group in MIT, led by Greitzer and in University of Maryland, led by Abed had been strongly supported in the past by AFOSR. The Moore-Greitzer model laid solid foundation for the use of feedback control for suppressing rotating stall and surge. The analytical low order state-space model derived in [25] captures the characteristics of rotating stall and surge, and used in both MIT [26, 27] and University of Maryland [22] to tackle rotating stall control. This work is further pursued by the research group in Georgia Institute of technology, led by Nett [6, 9], and by PRET led by Kokotovic [21]. It is interesting to note that classic bifurcation theory provided a powerful tool for both analysis and synthesis of rotating stall control. The bifurcation analysis yields a nonlinear feedback controller proposed in [22] that stabilizes the critical operating point, and later it is experimentally validated in [6, 9].

Our research program has focused on bifurcation approach to rotating stall and surge control using the low order Moore-Greitzer model. This is a continuation of the existing work in compressor control, and has potential to make contributions to nonlinear robust control. It should be clear that the difficulty associated with compressor control is due to the lack of corresponding theory and practice for bifurcation control, by the fact that rotating stall and surge are both phenomena of nonlinear bifurcations. Very few results are available for bifurcation control except those in [1, 3, 18] where state feedback is employed for bifurcation stabilization. Moreover the success in nonlinear

control of rotating stall as reported in [22, 6, 9] is based on bifurcation theory. These considerations motivated us to adopt a bifurcation control methodology to compressor control. A more profound reason for using bifurcation approach is due to nonlinear robust control. In the past, nonlinear control has focused on extension of linear control theory and design to nonlinear systems. The current trend in nonlinear robust control follows the same path. However nonlinear systems have their unique features that do not exist for linear systems. Simple generalization of linear control theory to nonlinear systems may not work. This is especially true for bifurcated systems which involve uncertain parameters. At critical values of the uncertain parameters, more than one equilibria are born at which stability changes. Often the critical modes of the linearized control systems are uncontrollable, or unobservable, or both. This is where linear control theory fails, which is exactly the rotating stall control problem faces. The development of bifurcation control theory is clearly an important part of nonlinear robust control, and has no parallel in linear robust control. Thus the bifurcation approach to compressor control problems will advance our knowledge to bifurcation stabilization and nonlinear robust control as well.

Research Results

The schematic compression system is shown in the following figure:



Schematic of compressor showing nondimensionalized lengths.

The total pressure at the upper stream of the compressor is denoted by p_T. The air flows through inlet guide vanes that straighten the flow. The compressor acts like an actuator that raises the pressure of the flow at the back of the compression system. The purpose of the compression system is to generate the required pressure rise which is the pressure difference between p_S and p_T. The established pressure rise is then used to provide the thrust for the jet airplane. Hence the compression system is the heart of the aeroengine. The ultimate objective of our research program is the improvement of the aeroengine performance.

In the past two years, our research program has focused on rotating stall and surge control that

are essential to compressor performance. Classic bifurcation theory was employed to analyze the problems of rotating stall and surge, and to obtain the feedback controllers that stabilize the critical operating points and enlarge the operating range of the compression system based on low order Moore-Greitzer model. Our research results were reported in a series of papers, and are summarized as follows.

- Bifurcation stabilization with output feedback.

  As mentioned earlier, rotating stall and surge controls are closely connected with bifurcation stabilization, because rotating stall corresponds to subcritical pitchfork bifurcation and surge corresponds to Hopf bifurcation. Stabilization of nonlinear control systems with smooth state feedback control has been studied by a number of people [2, 1, 3, 5, 29]. An interesting situation for nonlinear stabilization is when the linearized system has uncontrollable modes on imaginary axis with the rest of modes stable. This is so called *critical cases* for which the linear theory is inadequate. It becomes more intricate if the underlying nonlinear system involves a real-valued parameter. At critical values of the parameter, linearized system has unstable modes corresponding eigenvalues on imaginary axis, and additional equilibrium solutions will be born. The bifurcated solutions may, or may not be stable. The instability of the bifurcated solution may cause "hysteresis loop" in bifurcation diagram for both subcritical pitchfork bifurcation and Hopf bifurcation [16], and induce undesirable physical phenomina. This is exactly the case of rotating stall in axial flow compressors. Hence bifurcation stabilization is an important topic in nonlinear control.

  While most of the existing work in the open literature considers only state feedback for bifurcation stabilization, compressor control systems employ output feedback because often some of the state variables are not measurable, or too expensive to measure. It is thus necessary to investigate bifurcation stabilization for the case when only output measurements are available, and study the stabilizability property for various bifurcated systems. Our program focused our work on bifurcation stabilization. Specifically, the nonlinear system under consideration has single-input/single-output, and it involves a single parameter. At the critical value of the parameter, the linearized system possesses either a simple zero eigenvalue, or a pair of imaginary eigenvalues, and the bifurcated solution is unstable. Output feedback stabilization via smooth local controllers was studied for both stationary and nonstationary bifurcation. Two results were established for bifurcation stabilization. The first one was stabilizability conditions for the case where the critical mode is not linearly observable through output measurement. It was shown that nonlinear controllers do not offer any advantage over the linear ones for bifurcation

stabilization. The second one was stabilizability conditions for the case when the critical mode is linearly observable through output measurement. It was shown that linear controllers are adequate for stabilization of transcritical bifurcation, and quadratic controllers are adequate for stabilization of pitchfork and Hopf bifurcations, respectively. The proofs are constructive. Thus the results in this paper can be used to synthesize stabilizing controllers, if they exist.

- Nonlinear feedback for rotating stall control.

  A nonlinear feedback control law was proposed for rotating stall control. This feedback control law is different from that of [22] in that no distributed sensors are required, and output measurement was chosen as pressure rise that is a lumped parameter. This is important as distributed sensors such as hot wire for flow rate measurements are expensive and delicate, while pressure transducers are more durable to volatile flow field. This was the starting point for considering feedback control law of the form

  $$u(t) = \frac{K}{\sqrt{\Psi}}$$

  where Ψ denotes the pressure rise. The proposed control system employed pressure rise as output measurement and throttle position as actuating signal for which both sensor and actuator exist in the current configuration of axial flow compressors, and are lumped in nature that is contrast to other control method that employs either distributed actuators, or distributed sensors, or both.

  It should be emphasized that our results were obtained entirely with classic nonlinear bifurcation theory. This is due to the fact that linear control theory fails to apply to the bifurcated systems such as rotating stall and surge in compression system. Classical bifurcation analysis for nonlinear dynamics was used to derive a nonlinear feedback control law that eliminates the hysteresis loop associated with rotating stall and to extend the stable operating range in axial compressors. The stability of the critical operating point for controlled compressor was established using the center manifold theorem. Although our results were primitive and no advanced bifurcation control developed in [3, 1] was used, it yielded similar results as in [22]. More importantly, the use of pressure rise as output measurement also gives the opportunity for surge control. Recall that pure surge dynamics is governed by differential equations of flow rate and pressure rise, but not the amplitude of the disturbance flow. Hence the stabilizing control laws of [22] can not work for surge control. Our program is currently investigating the possibility of surge control with the same feedback control law.

- Linear and nonlinear feedback laws for rotating stall control.

Motivated by our early results we proposed several different feedback control laws where output measurements can be either pressure rise or averaged flow rate. Both linear and nonlinear feedback control laws were investigated that yielded similar results for rotating stall control. The foundation of the paper lies in those results established in our work for bifurcation stabilization. Specifically, the results from our work on nonlinear bifurcation stabilization were applied to rotating stall control that gave new stabilizing feedback controllers. It should be clear that the challenge to the proposed control system is that the critical mode of the linearized system corresponding to rotating stall is neither controllable nor observable. Both linear and nonlinear feedback control laws were proposed and shown to be effective in elimination of the hysteresis loop associated with rotating stall and in extension of the stable operating range of the axial flow compressor.

Although the results in this paper were applications of those results on bifurcation stabilization, it has several interesting points. First, it related rotating stall control to equivalent bifurcation stabilization, that was also studied in [3, 1]. Hence bifurcation stabilization can be used to synthesize stabilizing controllers for rotating stall control. Second, it indicated the stability ranges for different feedback controllers, and these ranges are finite. Moreover it is possible that the stabilizing ranges of the feedback gains can be zero for some of the compressor control systems, and thus stabilizing controllers do not exist in some cases. Fortunately stabilizing controllers do exist for practical compressor control systems such as the one at MIT.

- Further results on rotating stall control.

  In compressor control with throttle as actuators, an important consideration is that the operating point is different from the critical point of bifurcation, and that the throttle has to be positive due to the physical constraint. This problem was also investigated in our work where sensor signals are averaged flow rate on the circumference of the compressor or the pressure rise. Sufficient conditions were derived for the control law gains to guarantee that the subcritical pitchfork bifurcation responsible for hysteresis is rendered supercritical and that the the bifurcated solution is asymptotically stable. The proposed control laws gave practical solution for rotating stall in axial flow compressors. The numerical examples showed that the transformation of the bifurcation from subcritical to supercritical and the elimination of the hysteresis region.

- Bifurcation based surge control.

  With the success of our work on rotating stall control based on low order Moore-Greitzer model, our program moved quickly to surge control for axial flow compressors. Although there exist a

family of state feedback laws which stabilize the nonaxisymmetric equilibria near the operating point and eliminate the hypothesis induced by rotating stall, Hopf bifurcation associated with surge still exists under rotating stall control laws. Our research work in this regard introduced test functions whose zeros are critical to Hopf bifurcation for the closed-loop system where nonlinear feedback control laws are employed. These test functions were given in compact form. A particular test function was also developed to determine stability of the periodic solutions born at Hopf bifurcation. The analysis based on these test functions led to a new method of feedback design for control of both stationary and Hopf bifurcation in axial flow compressors. With our proposed control laws, feedback controllers can be designed to meet several bifurcation control requirements, including elimination of the behavior of surge, coupled with rotating stall. This is a significant result because in engineering practice, rotating stall and surge are often coupled that is called classic surge.

The success of this research program is inseparable from the control group in Wright-Patterson Air Force Base (WPAFB), led by Dr. Siva Banda. In fact almost all the results summarized in this report were the consequences of collaboration with the control group in WPAFB, including Dr. Andrew Sparks, and Dr. Siva Banda, and Mr. Paul Blue. Hence our research group is extremely grateful to the control group of Dr. Banda, and looking forward for further collaboration in the near future.

3 Personnel Supported

This research grant supported the PI for the summers of 1995, 1996, and 1997. It also supported partially one Ph.D student, and M.S. student. The details are as follows.

- Principle Investigator: Guoxiang Gu

  The PI has focused his work on both modeling and control of uncertain systems, and on bifurcation based compressor control. He was supported with four week salary for the summer of 1995, two month salary for the summer of 1996, and one and half month salary for the summer of 1997.

- Graduate Research Assistant: Gisoon Kim

  Gisoon's work focused on identification and control of uncertain systems. He was supported as 12.5% research assistantship for the academic year of 1994, matched with 37.5% teaching assistantship from the Department of Electrical Engineering, LSU. He completed his Ph.D Dissertation in May 1995.

- Graduate Research Assistant: Hyungun Song.

  He was a M.S. student working in the area of robust identification when he was supported as a graduate assistant with 25% RA. He completed his M.S. degree at the end of 1996, and then passed qualifying exam. He is now a Ph.D candidate, and continues his work on identification and control of linear uncertain systems.

4 Publications

Our research program in the past three years produced 9 journals publications, 2 book chapters, and 14 conference papers, including those accepted for publications. The PI also has a book in preparation for submission for publication in the research area of robust identification. These papers and publications are grouped into three parts according to their problem areas.

I. Modeling of Uncertain Systems

1. J. Chen, G. Gu and C. N. Nett, "Worst case identification of continuous-time systems via interpolation," *Automatica*, vol. 39, 1825-1837, Dec. 1994.

2. G. Gu and P. P. Khargonekar, "Identification in frequency domain," B. A. Francis and A. R. Tannenbaum eds., Lecture Notes in Control and Information Sciences, vol. 202, pp. 99-113, Springer-Verlag, London, UK, 1995.

3. P. P. Khargonekar, G. Gu, and J. Friedman, "Identification in H_∞: Theory and Applications," in Identification, Adaptation, Learning, S. Bittanti et al. eds., NATO ASI Series F: Computer and Systems Science, vol. 153, pp. 139-161, Spring er-Verlag, 1996.

4. G. Gu, C.-C. Chu, and P. P. Khargonekar, "From frequency domain data to state-space model," *Proc. of IEEE Conf. on Dec. and Contr.* (New Orleans, LA, Dec. 1995), pp. 1252-1257.

5. G. Gu, H. Song, and K. Zhou, "An integrated approach to modeling and control of flexible structures via $\mathcal{L}_\infty$ loopshaping," to appear in *Proc. of 36th IEEE Conf. on Dec. and Contr.*, 1997.

6. G. Gu, "Modeling of normalized coprime factors with H_∞ norm bounded uncertainty," *Proc. of IEEE Conf. on Dec. and Contr.* (Orlando, FL, Dec. 1994), pp. 3955-3960. Also under review for *IEEE Trans. Automat. Contr.*, submitted in 1996.

7. G. Gu, J. Chen, *Robust Identification and Model validation in $\mathcal{H}_\infty$*, book, under preparation.

II. Control of Uncertain Systems

1. G. Gu, J. Chen, and E. B. Lee, "Parametric $\mathcal{H}_\infty$ loop-shaping and weighted mixed sensitivity minimization," to appear in *IEEE Trans. Automat. Contr.*, 1997.

2. G. Gu, L. Qiu, "Connection of multiplicative or relative perturbation in coprime factors and gap metric uncertainty," to appear in *Automatica*, 1997.

3. J. Chen, G. Gu, C. N. Nett and D. Xiong, "A canonical structure for constrained optimal control problems," *Int. Journal on Robust and Nonlinear Control*, vol. 6, 727-741, 1996.

4. G. Gu, "Model reduction with relative/multiplicative error bounds and relations to controller reduction," *IEEE Trans. Automatic Control*, vol. AC-40, 1478-1485, Aug. 1995.

5. J. Chen, G. Gu and C. N. Nett, "A new method for computing delay margins for stability of linear delay systems," *Syst. and Contr. Letters*, vol. 26, 107-117, 1995.

6. G. Gu, J. Chen, and O. Toker, "Computation of $L_2[0, h]$ induced norms," *Proc. of 35th IEEE Conf. Dec. and Contr.* (Kobe, Japan), 4046-4051, Dec. 1996.

7. J. Chen, G. Gu, and K. Zhou, "Balanced truncation with relative/multiplicative error bounds in L_∞ norm," *Proc. of IEEE Conf. on Dec. and Contr.* (New Orleans, LA, Dec. 1995), pp. 3086-3091.

8. G. Gu, J. R. Cloutier, and G. Kim, "Gain scheduled missile autopilot design using observer-based H_∞ control," *Proc. 1995 Amer. Contr. Conf.* (Seattle, WA, June 1995), pp. 1951-1955.

9. G. Gu, J. Chen, and E. B. Lee, "Multivariable feedback control system design with H_∞ loop-shaping," *Proc. 1995 Amer. Contr. Conf.* (Seattle, WA, June 1995), pp. 2369-2373.

III. Bifurcation Based Compressor Control

1. X. Chen, G. Gu, P. Martin, and K. Zhou, "Bifurcation control with output feedback and its applications to rotating stall control," accepted by *Automatica*, 1997.

2. G. Gu, S. Banda and A. Sparks, "An overview of rotating stall and surge control for axial flow compressors," *Proc. of 35th IEEE Conf. on Dec. and Contr.* (Kobe, Japan) 2786-2791, Dec. 1996.

3. G. Gu, X. Chen, A. Sparks, and S. Banda, "Bifurcation stabilization with local output feedback," submitted to *SIAM J. Optimiz. and Contr.*, 1997. See also *Proc. of 1997 Amer. Conf.*, (Albuquerque, NM), 2193-2197, 1997.

4. W. Kang, G. Gu, A. Sparks, and S. Banda, "Surge control and test functions for axial flow compressors," *Proc. of 1997 Amer. Contr. Conf.*, (Albuquerque, NM), 3721-3725, 1997. It is also accepted by *Automatica*, 1997.

5. G. Gu, A. Sparks, and S. Banda, "Bifurcation based nonlinear feedback control for rotating stall in axial flow compressors," *Proc. of 1997 Amer. Contr. Conf.*, 1524:1528. It is also accepted for publication by *Int. J. Contr.*, 1996.

6. A. Sparks and G. Gu, "Control of compressor rotating stall without distributed sensing using bifurcation theory," *Proc. of 1997 Amer. Contr. Conf.*, (Albuquerque, NM), 3716-3720, 1997.

5 Interactions/Transitions and Others

a. The PI has given numerous presentations in major control conferences, and been invited to give several seminars in different research institutes. Because conference presentations can be found in the publication section, only those invited seminars in the past three years are listed below:

   (1) G. Gu, "Bifurcation stabilization and compressor control," College of Engineering, University of California at Riverside, June 1997.

   (2) G. Gu, "Bifurcation stabilization and applications to rotating stall control for axial flow compressors," Department of Mathematics, Naval Post Graduate School, Monterey, CA, July 1996.

   (3) G. Gu, "Modeling of coprime factors with gap metric uncertainty," Wright-Patterson AFB, Dayton, Ohio, July 1995.

b. In the past year, the PI collaborated closely with the control group of Dr. Siva Banda in WPAFB. Two groups visited each other at least twice a year, and had several joint papers in 1997 American Control Conference. Some of them were accepted by technical journals in control. See the section of publications. The PI also pro-actively seeks the partnership with other researchers in the same field, including Dr. Jim Paduano at MIT. Our program is currently in the process of collaborating with MIT research group on identification of acoustic modes for high order compression systems which will be an important move for us to enhance the application part of our research program.

c. Technology Transitions.

Performer:	Professor Guoxiang Gu, Louisiana State University.
Telephone:	(504) 388-5534.
Customer:	Compression System Component Center, Pratt & Whitney
Contact:	Dr. Carl N. Nett, Director
Telephone:	(937) 255-8682
Result:	Bifurcation stabilization; Active control of rotating stall and surge for Moore-Greitzer model to enlarge stable operating range.
Application:	Design of active feedback control laws to suppress rotating stall and surge, and to improve the performance of axial flow compressors.

d. New discoveries, inventions, or patent disclosures: None.

e. Honors/Awards: None.

6 Conclusion

During the past three years, our research program has accomplished more than it was initially planned. Our achievements include modeling and control of uncertain systems as proposed that is theory oriented, and compressor control which was not planned in our proposal that is application oriented. The research results were reported in 9 journal publications, 2 book chapters, and 14 conference papers, and were summarized in this final report. The PI has a book in the process of submission for publication in the area of robust identification. For the past three years, two graduate students were also trained, one at Ph.D level, and one at M.S. level who graduated with degrees. Several other graduate students were trained through various research projects under this program, and through course work with special topics in these two research areas. These students are in the process of completing their Ph.D Dissertations and M.S. Theses with one of them expected to graduate next year with Ph.D degree specialing in compressor control and nonlinear robust control. Considering the size of our program and limited amount of funding, this is a quite accomplishment for us in both research and in education. Our program was very fortunate to work under the guidance of Dr. Jacobs during our three year period, and to have a strong tie with the control group in Wright Laboratory at Wright-Patterson Air Force Base led by Dr. Siva Banda. With the preparation of this research program and close collaboration with the control group at Wright-Patterson Air Force Base led by Dr. Siva Banda, we are well positioned to accomplish research objectives for rotating stall and surge control that is in the interest of Air Force. Our control group at LSU is confident that our research program has the capability to contribute further to the DoD mission in the near future.

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